urements of the time between successive echoes made over periods of weeks. They are rounded off to the last significant figure and the tolerances represent the larger of the fluctuations in these or the accuracy of a specific measurement. Transit-time errors attributable to the transducer, determined after all the velocity data were completed, are about 1%. They are not applied because (1) they could not be systematically obtained, (2) except for v_9's they are less than the over-all velocity tolerances specified for each velocity, and (3) we have no information on the fluctuations in the transit-time correction measurements themselves. Taken at face value, a 1% average correction to the velocities would scale the antimony stiffness values by 2%.

Before the numerical evaluation of the constants was carried out, the general features of the velocity data were examined for consistency with the equations of Table I as follows: v_{10} being greater than v_{13} clearly fixes c_{14} as positive for the axes senses chosen. In turn, this requires that $v_{12}^2+v_{14}^2$ be greater than $v_9^2+v_{11}^2$; $v_2^2>v_{10}^2$; $v_{13}^2>v_3^2$; and $v_{12}^2>v_9^2$, which is indeed the case within experimental error. These inequalities are compatible with assigning the larger velocity value of two coupled modes, normally associated with the longitudinal mode, to the positive radical of the relevant expressions, i.e., in the pairs v_2 and v_3, v_4 and v_6, v_9 and v_{11}, and v_{12} and v_{14}, the first velocity is the greater one. Next, the eight redundancy relations, a more sensitive and detailed test of the data than the trace relations used by ELR, were evaluated; one obtains that $v_{11}=1.25\pm1\%$ for antimony is incompatible with the others in this formalism. Consequently, attempts to fit to it and its inclusion in a least-squares function are meaningless and it is ignored in our calculation of antimony's constants. A possible reason for v_{11}'s incompatibility is discussed in the section on elastic-wave refraction.

Generally stated, our least-squares procedure is based on adjusting each of the 14 squares of the velocities within experimental error so that they give a minimum deviation from the central experimental-velocity-squared values and, when inserted in Eqs. (1) through (14), yield a common value for each of the six stiffness constants.

The least-squares function used is

$$F=\sum_{i=1}^{14}\left(\frac{v_{ia}^2-v_{io}^2}{2v_{io}\Delta v_i}\right)^2,$$

where the subscripts a and o signify adjusted and observed, and Δv_i is the experimental uncertainty in the ith velocity. This task is simplified by initially selecting those velocities and combinations of velocities which are related to the smallest number of stiffness constants and then extending the selection in steps to include more and more velocities until all the constants are obtained. As more velocities are included, the previously obtained values are readjusted when necessary. Specifically, first v_5^2, v_8^2, $v_{10}^2+v_{13}^2$, and $v_2^2+v_3^2$ are adjusted and c_{44} and c_{66} obtained. With these values and $v_{10}^2-v_{13}^2$ and $v_2^2-v_3^2$, a common value for c_{14} is obtained, usually upon readjustment of the previously obtained velocities and constants. After this, c_{11} is similarly obtained but from v_1^2, $v_4^2+v_6^2$, and $(v_4^2-v_6^2)^2$, and c_{33} from v_7^2, $v_9^2+v_{11}^2$, and $v_{12}^2+v_{14}^2$. Finally c_{13} is obtained from $(v_9^2-v_{11}^2)^2$ and $(v_{12}^2-v_{14}^2)^2$, again readjusting the already obtained values as necessary. Because each of the functions from which c_{13} is calculable yields two values, the common one is, of course, the proper one. (Antimony calculations involving incompatible v_{11} are omitted.)

The results of this procedure for antimony and for the complete bismuth data of ELR, and the results of other workers and their procedures, are presented in Tables II, III, and IV. These are next discussed.

VI. DISCUSSION

A. Nature and Limitations of Fit

In the course of fitting the antimony data, it became clear that the 14 equations of Table I intersect in a well-defined region of a 6-dimensional stiffness-constant space and that only a very narrow range of values for the constants is possible. The bounding limits of this region are, roughly, such that a change greater than 5% in almost any constant appears sufficient to bring one or more of the 14 velocities outside the experimental range. Accordingly, the basis for choosing the constants

Table II. Elastic stiffness constants at room temperature.

	c_{11}	c_{12}	c_{13}	c_{14}	c_{33}	c_{44}	c_{66}	Source
Sb	99.4(1)	30.9(1)	26.4(4)	+21.6(4)	44.5(9)	39.5(5)	34.2(5)	This work, least squares
	99.31				44.59			Eckstein,[a] transmission technique at 77°K
	81.00	11.00		+18.00	43.60	33.60	35.00	Leventhal,[b] echo technique
	79.20	24.70	26.10	+11.00	42.70	28.50	27.30	Bridgman,[c] static technique
Bi	63.22	24.42	24.40 ±.09	+ 7.20	38.11	11.30	19.40	ELR,[d] least-squares recalculation
	63.50	24.70	24.50	+ 7.23	38.10	11.30	19.40	ELR,[d] transmission technique
			21.50	+ 7.20				Kor's,[e] recalculation of ELR
	62.90	35.00	21.10	− 4.23	44.00	10.84	13.37	Bridgman,[c] static technique

Units: 10^{10} dyn/cm^2.

[a] See Ref. 10. [b] See Ref. 13. [c] See Ref. 2. [d] See Ref. 1. [e] See Ref. 14.